Covariant Foundation Theory

Richard Mould*

Abstract

A foundation theory of quantum mechanics is proposed in another paper that governs state reduction in general, and particle localization in particular. The theory is based on a set of four rules called the nRules. In the present paper the nRules and the nRule equations are shown to be Lorentz invariant. The details of a non-local, space-like collapse of a relativistic wave function depend on the observer, but the form of the collapse is determined by the invariant nRule equations.

Introduction

The nRules are a set of four rules that were previously said to describe all non-relativistic individual quantum mechanical processes [1]. These rules are listed in the appendix. They generate equations that characterize solutions to Schrödinger's equation, describing their direction and influence. For instance, the capture of a particle by a detector is given by the nRule equation

$$\Phi(t \geq t_0) = \psi(t)d_0(t) + \underline{d}_1(t) \tag{1}$$

where the first component includes the free particle $\psi(t)$ and the detector $d_0(t)$ prior to capture. The second component $\underline{d}_1(t)$ is the detector after it has captured the particle. It is equal to zero at time t_0. Probability current flows from the first component to the second by an amount determined by the Schrödinger equation; so the square modulus of the first component decreases as the second one increases. According to the nRules, the second one is threfore subject to a stochastic hit followed by a state reduction; and when that happens the first component goes to zero. An underlined component like $\underline{d}_1(t)$ is called a "ready" component, which means that it is subject to a stochastic hit of this

*Department of Physics and Astronomy, State University of New York, Stony Brook, New York 11794-3800; http://ms.cc.sunysb.edu/~rmould

kind. Evidently the nRules describe the route taken by an individual quantum mechanical system when it is *driven* by the Schrödinger equation and *reduced* by a stochastic event. The rest of the universe is included in each component but is not shown. Full details are given in Ref. 1.

The nRules are claimed to apply to microscopic as well as macroscopic processes. A microscopic example is the decay of a free neutron given by

$$\Phi(t \geq t_0) = n(t) + \underline{e}p\overline{\nu}(t) \tag{2}$$

where again the second component is zero at t_0 and increases in time because of Schrödinger produced current flow from the first component to the second. Underlining a single state like $\underline{e}$ means that the entire component is "ready", so a stochastic hit on $\underline{e}p\overline{\nu}(t)$ results in a state reduction in which only that component survives. Under these rules state reduction can occur in any system, large or small – not just one that interacts with a macroscopic measuring device or outside observer. See Ref. 1 for details concerning Eq. 2.

Relativistic Collapse

Equations 1 and 2 also apply relativistically. The dynamical principle determines when and how much probability current flows from one component to another in a given Lorentz frame. In addition, the energy and momentum distribution of each particle in each component is also a function of the Lorentz frame. However, the components appearing as they do in these equations (satisfying the condition of completeness in nRule 1) are the same for all observers. So whether or not the dynamical principle is relativistic, the above equations (Eqs. 1 and 2) are correct. They become nRule equations when the ready components are identified, and when they are made subject to the collapse protocol established by the nRules.

To the extent that the components in these equations are locally interacting, nothing beyond them is affected by a collapse of the wave. However, there may be non-local correlations between the states that spreads the collapse over finite regions of Minkowski space. Suppose that two particles p_1 and p_2 are correlated in the spin zero state

$$\Psi(p_1, p_2) = p_1(\uparrow)p_2(\downarrow) - p_1(\downarrow)p_2(\uparrow) \tag{3}$$

A state function need not be normalized because probability currents, not functions, are normalized under the nRules. Suppose the first particle is measured at an event A and found to have a spin-up, and the second particle is measured at an event B and found to have spin-down.

If the pair in Eq. 3 is created by the decay of a composite particle p_c, the corresponding nRule equation will be given by

$$\Phi(t \geq t_{00}) = p_c(t) + \underline{\Psi}(p_1, p_2, t)$$

where the second component is zero at $t_{00} = 0$ and increases in time. The variable t is the time in the given Lorentz frame. Probability current will flow from the first component to the underlined 'ready' component $\underline{\Psi}$, leading to a stochastic hit and localized reduction at time t_0 given by

$$\Phi(t \geq t_0 > t_{00}) = \Psi(p_1, p_2, t)$$

which is Eq. 3 given as a function of time.

When the spin-measuring devices M_1 and M_2 of particle p_1 and p_2 are introduced we have the nRule equation

$$\begin{aligned}\Phi(t \geq t_0) = \Psi(p_1, p_2, t) \otimes M_1 M_2 \quad &+ \quad [p_1(\uparrow, t)\underline{M}_1]p_2(\downarrow, t) \otimes M_2 \\ &+ \quad p_1(\uparrow, t)[p_2(\downarrow, t)\underline{M}_2] \otimes M_1 \\ &+ \quad [p_1(\downarrow, t)\underline{M}_1]p_2(\uparrow, t) \otimes M_2 \\ &+ \quad p_1(\downarrow, t)[p_2(\uparrow, t)\underline{M}_2] \otimes M_1\end{aligned} \tag{4}$$

where both measuring devices are on standby in the first component of Eq. 4, and the four 'ready' components (on the right) are zero at t_0 and may increase in time. In the ready component of the first row, the first particle engages the spin-measuring device M_1 (square brackets), and in the second row the second particle engages M_2. The third and fourth rows are similar except that they provide for the reverse spin measurements. In the following, the third and fourth row components are ignored.

Event A First

The first case to be considered is one in which p_1 is at rest. Event A (in which p_1 engages M_1) is assumed to occur before event B in this Lorentz frame. We therefore ignore the second row as well as the third and fourth in Eq. 4.

The first component of Eq. 4 will remain undiminished until just before M_1 is engaged at time t_A (the time of event A) since probability current does not flow until that time. This is shown in the Minkowski diagram of Fig. 1a where $c = 1$. As soon as the interaction begins, current will flow into the ready

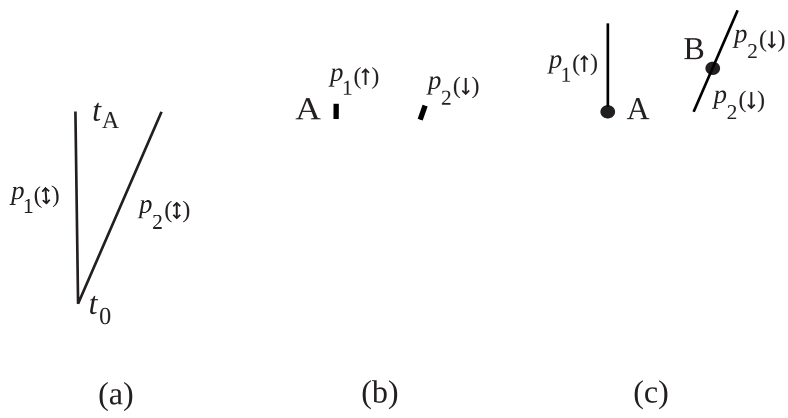

Figure 1: Component diagrams for event A reduction

component in the first row. This component will be zero until that moment but will grow rapidly thereafter. The brief interaction lifetime appears in Fig. 1b during which event A is stochastically chosen, so the first particle goes spin-up at the *same time* the second particle goes spin-down as mandated in the ready component in the first row of Eq. 4. Other components in Eq. 4 go to zero at this time. The second measuring device M_2 is still unengaged.

The nRule equation following this collapse is shown in Fig. 1c and is given by

$$\Phi(t \geq t_A > t_0) = [p_1(\uparrow, t)M_1]p_2(\downarrow, t) \otimes M_2 \rightarrow [p_1(\uparrow, t)M_1][p_2(\downarrow, t)M_2] \qquad (5)$$

where the second term is *not* a ready component since the measurement of the second particle at event B does not result in a discontinuity of any kind. Discontinuity in some variable is required of a ready component by nRule (1). The spin of p_2 is simply confirmed by that measurement, so there is no stochastic choice to be made. This means that Eq. 5 evolves continuously and classically, where the two terms in that equation are parts of a single component. One passes continuously into the other as indicated by the arrow. This passage occurs in the brief time during which the particle engages the measuring device M_2, so the first part of Eq. 5 goes quickly into the second part at time t_B in this frame.

The collapse in Fig. 1 is certainly not invariant. That is not required. It is required that the nRules and the resulting nRule equations (Eqs. 4, 5, and 6) that govern a collapse are invariant. They are the same for all observers as will be shown.

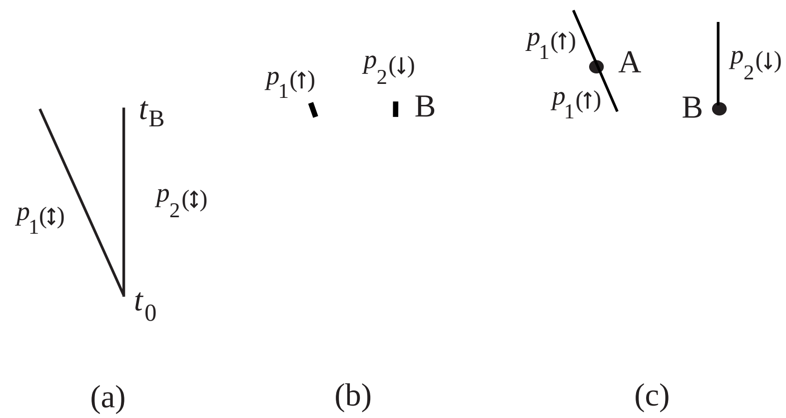

Figure 2: Component diagrams for event B reduction

Event B First

The second case to be considered is one in which p_2 is at rest. Event B (in which p_2 engages M_2) is assumed to occur before event A in this Lorentz frame. We now ignore the ready component in the first row as well as the third and fourth rows in Eq. 4.

This case is shown in Fig. 2, where Fig. 2a is the evolution of the particles prior to a stochastic hit. As before, both particles have an uncertain spin during that time. Probability current flows very briefly in Fig. 2b, leading to a stochastic hit on the ready component in the second row of Eq. 4. Continued evolution of the state after t_B (in this frame) is shown in Fig. 2c and is given by

$$\Phi(t \geq t_B > t_0) = p_1(\uparrow, t)[p_2(\downarrow, t)M_2] \otimes M_1 \rightarrow [p_1(\uparrow, t)M_1][p_2(\downarrow, t)M_2] \qquad (6)$$

where, as before, the second term is not a ready component inasmuch as the measurement of the first particle at event A does not result in a discontinuity of any kind. The spin of p_1 is simply confirmed by this measurement. The first term in Eq. 6 passes continuously into the second term as indicated by the arrow. That passage occurs during the brief time that the particle engages the measuring device M_1, so the first part of Eq. 6 goes quickly into the second part during the interaction time at t_A in this frame.

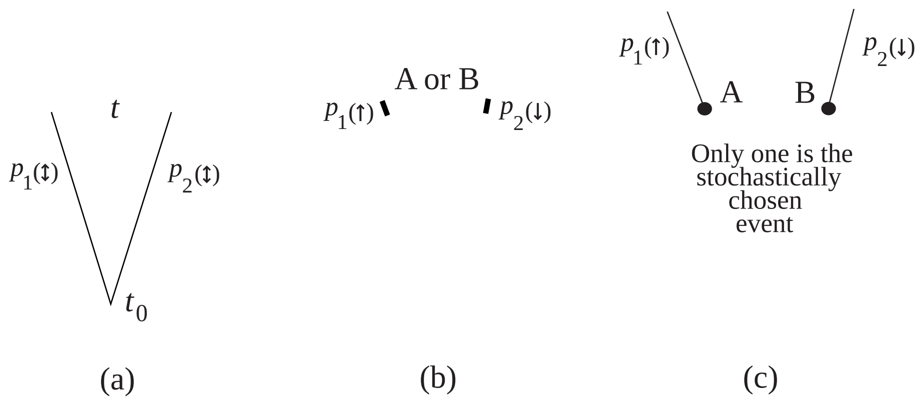

Figure 3: Component diagrams for symmetric reduction

The Symmetric Case

A third possibility refers to a Lorentz frame in which both particles separate from each other with equal velocities. Assume that events A and B are now ostensibly simultaneous. This is shown in Fig. 3

In this case both particles arrive at their respective measuring devices at the same time, so probability current flows equally into the ready components of the first and second rows of Eq. 4 (where again the third and fourth rows are ignored because we assume that p_1 measures spin-up). But event A or B, will not be exactly simultaneous. The first to occur depends on which component is the first to be stochastically chosen during the time of mutual current flow. If event A is chosen, event B will be eliminated from Eq. 4 because the second row will then go immediately to zero. The reduced state is then Eq. 5 where the first term goes immediately into the second term, so event B occurs in the reduced state rather than in the initial state. However, events A and B are “essentially” simultaneous in this frame – within the time Δt of the interaction.

Relativistic Invariance

The classic model of relativistic invariance is one in which a particular solution to the equation of motion dependents on the Lorentz observer. That solution follows from initial conditions that are peculiar to that observer. However, the equations of motion are the same for all observers. Invariance is therefore a property of the lawfully given equations of motion, not particular solutions of that equation.

This also holds for the rules that govern a collapse of the wave; that is, the nRules and nRule equations are invariant under Lorentz transformation, but the particular solution is different for different observers. We have considered a measurement as viewed by three observers in Figs. 1, 2, and 3, where all three are lawfully determined by the nRule equations. Prior to a collapse of the wave, all three of these interactions are governed by Eq. 4. After a reduction resulting in event A, the solution is governed by Eq. 5. And after a reduction resulting in event B, the solution is governed by Eq. 6. All three of these equations are derived from the nRules that are valid throughout for all observers. In all these cases the Minkowski diagram is copied directly from the governing nRule equation, allowing only for different initial conditions. Therefore, we say that: *The nRules and their nRule equations are Lorentz invariant.*

Inasmuch as these rules do not refer directly to the variables of any one coordinate observer, we are free to extend the above results beyond the Lorentz group. We claim more broadly that: *The nRules and the nRule equations are generally covariant.*

Correlations Preserved

Particle 2 goes spin down *before* event B in Fig. 1b, but it goes spin down *simultaneous* with event B in Fig. 2. Event B is an invariant reality that corresponds to the measurement of spin down; however, the 'time' at which particle 2 goes spin down is not the same for all observers. It is a function of the particular solution in the given Lorentz frame as required by the nRules applied to that frame. Different Lorentz observers differ about this time, where the claim of one observer cannot be objectively verified or disputed by any other observer. This disagreement reflects the 'up-down' correlation between the particles in a single component that holds at every moment in every frame during the time that that component exists in that frame. It has nothing to do with the nRules or the nRule equations as such, although it follows from these rules and their equations.

Particle Waves

In the above cases we considered point particles p_1 and p_2. We need to see what a relativistic reduction looks like when a location measurement is made on a particle wave that is spread out over space. In Fig. 4 a one dimensional particle wave is confined to the shaded area of width Δx. Two position measuring

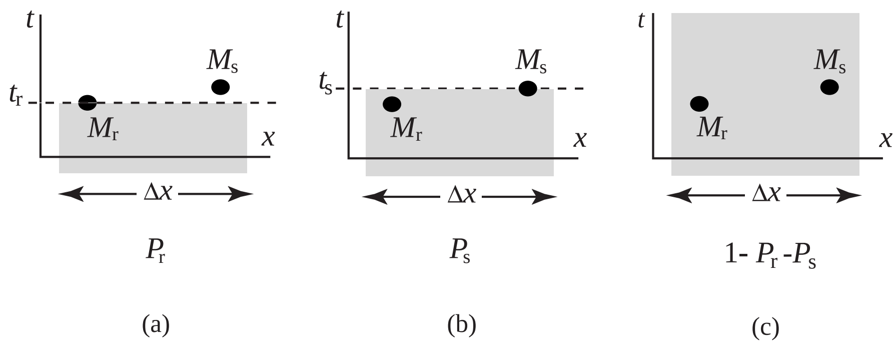

Figure 4: Position measurements of particle wave

devices M_r and M_s are activated at events M_r and M_s on the diagram, where the world lines of the devices are not otherwise shown. The nRule equation for these interactions is given by

$$\Phi(t \geq t_0) = \psi(t) \otimes M_r(t)M_s(t) + \underline{M}'_r(t) \otimes M'_s(t) + \underline{M}'_s(t) \otimes M'_r(t) \qquad (7)$$

where $\psi(t)$ is the incoming particle wave, and $M'_r(t)$ and $M'_s(t)$ are the measuring devices that have captured the particle. As always the ready components are initially zero and increase in time. The probability of capture by M_r is P_r and the probability of capture by M_s is P_s.

Figure 4a shows a capture by detector M_r at event M_r. That will happen when probability current flowing into the second component in Eq. 7 results in a stochastic hit. The collapse of the wave is shown in the figure to occur along the horizontal line of t_r, where Eq. 7 holds prior to that time. After collapse the nRule equation is given by

$$\Phi(t \geq t_r > t_0) = M'_r(t) \otimes M_s(t)$$

leaving the particle inside detector M_r.

The collapse at t_r excludes the possibility that the detector at M_s can be chosen, for that choice drives the third component to zero in Eq. 7. However, it really doesn't matter if the measuring device M_s is above or below the time line t_r, for if M_r is stochastically chosen with probability P_r, then M_s will not be chosen no matter were it is located.

If M_r is not stochastically chosen, then the third component in Eq. 7 may be selected with a probability P_s. This is shown in Fig. 4b. The wave collapse then takes place along the horizontal line t_s, after which the reduced nRule equation is

$$\Phi(t \geq t_s > t_0) = M'_s(t) \otimes M_r(t)$$

leaving the particle inside the detector M_s.

As before, the hit on M_s excludes the possibility of a second hit on M_r because the collapse will cause the second component in Eq. 7 to go to zero. And as before, it doesn't matter if the measuring device M_r is above or below the time line t_s; for if M_s is stochastically chosen, then M_r will not be chosen no matter were it is located.

Figure 4c shows the "no collapse" case when neither detector captures the particle.

The x'-axis of another Lorentz observer will cut through the origin in each of the above diagrams making an angle of θ with the x-axis, where $\tan\theta < 1$. For that observer the collapse takes place along lines t'_r or t'_s, both of which are parallel to x'. It will not matter to this observer if the stochastic hit on one of the detectors is above or below one of those lines for reasons similar to those given above. The only objective reality for either one of these observers is that the device M_r captures the particle with a probability P_r, the device M_s captures the particle with a probability P_s, and no capture occurs with a probability $1 - P_r - P_s$. The invariance of these probabilities must be guaranteed by the dynamical principle.

Conclusion

The collapse of a wave establishes a new boundary condition on the system. Apart from the dynamical principle and the boundary events themselves, the only true invariants in a relativistic collapse are the nRules that apply uniformly to all observers at all times, plus the associated nRule equation that applies *following the installation of any new* boundary condition. So Eq. 4 applies after the initial conditions in that equation, Eq. 5 applies after a new boundary is established at event A, and Eq. 6 applies after a new boundary is established event B.

Appendix

The following four nRules are those found in Ref. 1.

We define *ready components* to be the basis components of state reduction. These are the components that are chosen to survive the collapse of the wave function. They are underlined throughout. Components that are not ready are called *realized components* and appear without an underline.

The first nRule describes how ready components are introduced into the equations of motion.

nRule (1): *If an irreversible interaction produces a complete component that is discontinuous with its predecessor in some variable, then it is a ready component. Otherwise a component is realized.*

[**note:** A *complete component* is one that includes all the (anti)symmetrized objects in the universe. Each included object is itself complete in that it is not a partial expansion in some representation.]

The second rule establishes the existence of a stochastic trigger. The flow per unit time of square modulus is given by the square modular current J, and the total square modulus of the system is given by s.

nRule (2): *A systemic stochastic trigger strikes a ready component with a probability per unit time equal to the positive probability current J/s flowing into it. A realized component is not stochastically chosen.*

[**note**: The division of J by s automatically normalizes the system at each moment of time. Currents rather than functions are normalized under these rules.]

The collapse of a wave is given by nRule (3)

nRule (3): *When a ready component is stochastically chosen it will become a realized component, and all other (non-chosen) components will go immediately to zero.*

[**note**: We can amend nRule (3) so that other components do *not* go to zero. It does no harm to let them stand unchanged after a stochastic hit because there will be no further consequence. Square modulus has no physical meaning in the nRules, and current no longer flows into or out of these components because the Hamiltonian has already passed them by. The spent components would have the status of "phantoms" as defined in Ref. 1. The decision to let these not-chosen components go to zero or to let them stand is like the decision in standard quantum mechanics to renormalize (or not) after a measurement]

The fourth nRule has a less obvious meaning.

nRule (4): *A ready component cannot transmit probability current to other components or advance its own evolution.*

[**note**: The fourth nRule is enforced by withholding a ready component's Hamiltonian as explained in Ref. 1, thereby introducing a non-unitary intervention.]

References

[1] R. A. Mould "A Foundation Theory of Quantum Mechanics", quant-ph/0607063; *AIP Conf. Proc.* **844**, 256-271 (2006): "Are There Quantum Jumps? The Present Status of Quantum Mechanics", Trieste, Italy & Losinj, Croatia, Sept. 5-10, 2005